Lab 9

Simple Harmonic Oscillation

A. Purpose

To study the periodic oscillation of a spring-mass system.

B. Introduction

Oscillation is a periodic back and forth motion of some measure about a central value or between two or more different states. Not only in mechanical systems do oscillations occur, but in dynamic systems in most areas of science, e.g. the beating of the human heart, business cycles in economics, or vibrations of strings in guitar and other string instruments. This experiment studies some of the basic properties of simple harmonic oscillation by the behavior of a typical spring-mass system.

In Newtonian mechanics, the equation of motion for 1-D simple harmonic motion can be obtained through Newton's second law and Hooke's law, which give a second-order linear ordinary differential equation, called the equation of motion of the oscillator.

$$F_{net} = M\frac{d^2x}{dt^2} = -kx \Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{M}x = -\omega^2 x, \text{ where } \omega = \sqrt{\frac{k}{M}}, \tag{1}$$

where M is the inertia mass of the oscillating body, x is its displacement from the equilibrium (or mean) position, ω is the angular frequency and k is the spring constant. The solution to eq(1) is

$$x = A\sin(\omega t + \phi), \tag{2}$$

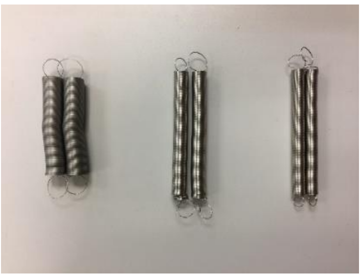
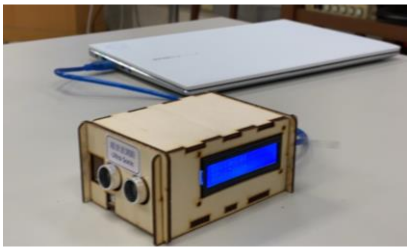
where A is the maximum of the displacement, called the amplitude and ϕ is the initial phase. The total energy (E_{tot}) for this undamped oscillator is conserved during the oscillation.

Note that in eq(1), the spring is assumed massless. If the spring instead has a finite mass m, the angular frequency should be modified by

$$\omega = \sqrt{\frac{k}{M + m/3}} \quad . \tag{3}$$

In reality, the amplitude of the motion would reduce with time due to "damping." (most of the time, due to friction). Damping is produced by processes that dissipate the energy stored in the oscillation. For example, viscous drag in liquid causes the oscillatory system to slow down, or the induced electric eddy currents in the system would dissipate the kinetic energy as heat. Usually, the viscous damping force is proportional to the speed of the oscillator, or to the squared speed of the oscillator.

C. Apparatus

			
Slide Cart and track	Springs of three different spring constants[1]	Arduino Case[2] + CoolTerm	Tracker

D. Procedures

1. Pre-lab assignments (hand in before the lab)
   (1) Download Tracker[3] and the driver for CoolTerm[4]
   (2) Make a flowchart of this experiment and answer the questions
      (i) Suppose the spring is evenly stretched with a cart attached. Prove eq(3).
      (Hint: divide the spring into n parts (as Fig. 1 shows), and write down the total energy of the system. Obtain its equations of motion by the time derivative of the total energy.)

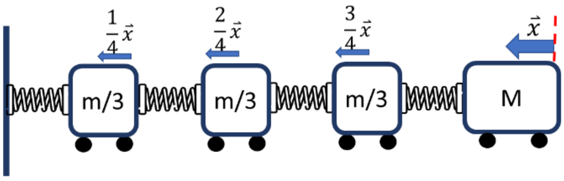


Fig. 1: The cart of mass M is attached to a spring of mass m. The spring is divided into three parts of the same mass with their speed shown above. In the question, you should divide the spring into n parts with the limit $n \to \infty$, where the speed of the part of the spring nearest to the cart is approximately $\dot{x}$.

(ii) Consider a simple harmonic oscillator under a constant friction force f. Find the equation of motion and the period of the oscillation. How would this friction force affect the amplitudes of the oscillation? **(Hint: work-energy theorem)**

(iii) Suppose the friction force is proportional to the speed of the oscillator, that is,

$$M\ddot{x} = -kx - b\dot{x}$$

where b is a constant called the "(viscous) damping coefficient."
If the oscillator has the initial position x_0 and the initial speed $\dot{x}_0$, find the general solution to the equation with the "underdamped" condition $b^2 < 4Mk$.

[1] During the experiment, to save time, we assume the springs of the same type have the same spring constant.
[2] There are two sensors on the case with the left one being the transmitter of the ultrasound and the right one being the receiver of the reflected waves.
[3] To install Tracker: https://physlets.org/tracker/
[4] To install CoolTerm: http://freeware.the-meiers.org

(iv) (Important!) See the following link for Tracker Tutorial:
https://www.youtube.com/watch?v=8EX16k_xTso

(v) (Important!) See the following link for fitting data by Matlab:
https://youtu.be/Tr3oTQpb-mM

2. In-lab activities

(1) Measurement of spring constants

(i) Static method

Obtain the static spring constant k_s for each kind of spring by using Hooke's law with at least five different hanging masses. Plot and fit the data by Matlab to obtain the final result $k_s = k_{s,best} + \delta k_s$. (Hint: the introductory class)

(ii) Dynamic method

Obtain the dynamic spring constant k_d for each kind of spring by measuring the period of the oscillating spring with at least five different hanging masses. Plot and fit the data by Matlab to obtain the final result $k_d = k_{d,best} + \delta k_d$. (Hint: measure the time for at least 20 complete cycles of the oscillation and use eq(3).)

(Question: What's more accurate: a single cycle, or several? Why?)

(iii) Compare the results obtained in procedures (i) and (ii). **Do you expect them to be the same? Are they the same? Explain in your post-lab report.**

(2) Conservation of energy in the simple harmonic oscillation

Record a video of ONE oscillating spring with a proper hanging mass. Analyze the video by Tracker. Calculate the mechanical energy of the oscillation for at least 10 points. Plot displacement vs. time and the mechanical energy vs. time. Explain the result you found in your report. (Hint: use the results of the previous part to determine which spring is more appropriate for this experiment.)

(3) Simple harmonic oscillation under constant friction force

Consider the setup as shown in Fig. 2. Choose two springs as a composition of one trial in this experiment. Two different trials are needed for this part.

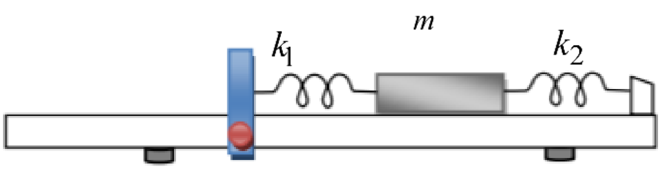


Fig. 2: Two springs, each of spring constant k_1 and k_2, are attached to a cart (with blue sides) of mass m as shown. If initially one moves the cart by a distance x_0, after the release, the cart would start oscillating back and forth. (k_1 and k_2 need not be different.)

(i) Calculate the theoretical values of oscillating periods of each trial.

(ii) Make the track horizontal by the level.

(iii) Use Arduino Case with CoolTerm to obtain the data

(a) Connect the Arduino Case to the laptop and run CoolTerm.

(b) Place the Arduino Case on the track and make sure the ultrasound transmitted by the case is perpendicular to the back of the cart on the track

(c) While the cart is moving, the data will be collected in CoolTerm.

(d) Disconnect Coolterm with the case and copy the data from CoolTerm to the datasheet in Excel to store it.

(e) Clear the data in CoolTerm so as to move on to the next trial

(iv) Find the oscillating periods and the effective friction force f of each trial by plotting the displacement x versus time t. (Hint: Q2 in the pre-lab.)

(v) Find another way to obtain the effective friction force f.

(vi) Compare the results you obtained above.

(vii) **(Bonus)** Obtain the average speed and the average displacement of the cart of all time and use the stacked area chart in Excel to plot total energy versus time.

(4) Damped oscillation

Use the cart with red sides to do this experiment. Also, it is recommended to use the two springs of the median spring constant in this part.

(i) Make the magnet under the cart the highest position

(ii) Follow the same procedures in the last part to obtain the data and the period of the oscillation.

(iii) Adjust the magnet under the cart to the lowest position and re-do the experiment

(iv) Change back to the cart with blue sides. Add masses on it until the total mass is about the same as the cart with red sides. Re-do the experiment and calculate the period of the oscillation this time.

(v) Compare the results and explain what you find in your post-lab report.

(vi) Fit the data via Matlab and compare with the theory. (Hint: "cftool" in Matlab)

(5) **(Optional)** Coupled Oscillation

Consider the setup as Fig. 3 shows. Move one cart away from its equilibrium with the other fixed. Release the two carts at the same time. Record a video of this oscillation. Use Tracker to analyze the motions of two carts and apply Fast Fourier Transform to the data. You will obtain two characteristic frequencies for this motion. These two frequencies correspond to the normal modes of the motion, which are symmetric mode and anti-symmetric mode respectively as shown in Fig. 4. Compare the results with the theory.

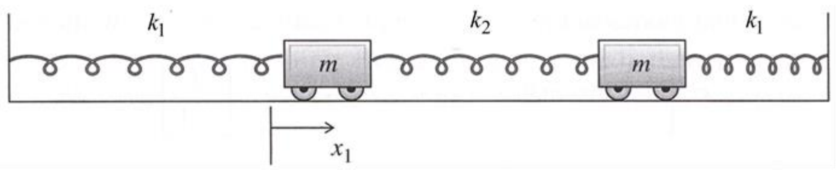


Fig. 3: Coupled Oscillation of two carts with three springs. Note the spring in the middle is different from those on the two sides and the two carts are of the same mass.

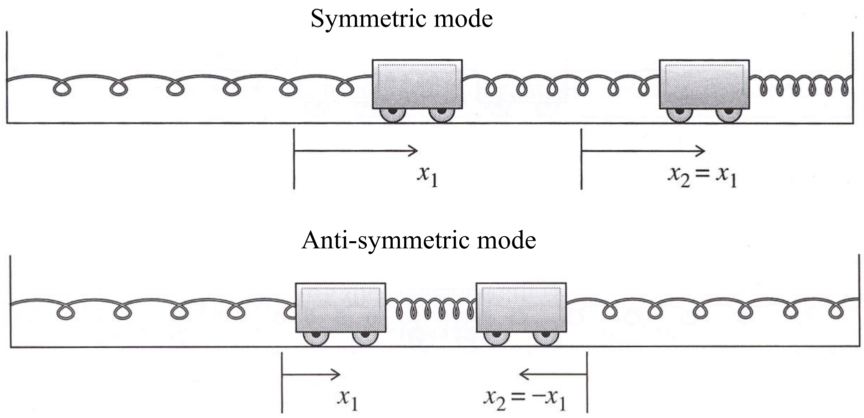


Fig. 4: Normal modes of two-cart coupled oscillation. The upper picture shows the symmetric mode where two carts move to the right by the same distance $x_1 = x_2$, and the lower one shows the anti-symmetric mode in which one cart move to the right by the distance x_1 while the other moves to the left by the same distance.

3. Post-lab report
   (1) Recopy and organize your data from the in-lab tables in a neat and more readable form.
   (2) Analyze the data you obtained in the lab and answer the given questions
   (3) Compare the results with the theory, and discuss the uncertainties that occur in the experiments, and how they influence the experiments. (Quantitatively, if possible.)

E. Questions

1. While deriving eq(3), we assume that the spring is evenly stretched. If the spring is not attached to the side of the cart but has a hanging mass under it, explain how you would modify eq(3) with gravity. Compare your modification with the experimental results.
2. At the beginning of the experiment, you are asked to make the track level. What would happen if the track is instead tilted by an angle θ? Should the results you obtained be modified? Explain by providing a physical reasoning.
3. Try to obtain the damping coefficients of the damped oscillations. Does the position of the magnet affect the results? Explain.
4. **(Optional)** Obtain the Fourier transform of damped oscillating data. Then fit the result by a Lorentzian curve to obtain the power spectrum of the experiment. Explain the result.

F. References

Easton, Don. "Simplifying the motion of coupled oscillators using the FFT." The Physics Teacher 44.1 (2006): 24-26.

Carnevali, Antonino, and Cynthia L. Newton. "Coupled harmonic oscillators made easy." The Physics Teacher 38.8 (2000): 503-505.

Rodriguez, Eduardo E., and Gabriel A. Gesnouin. "Effective mass of an oscillating spring." The Physics Teacher 45.2 (2007): 100-103.